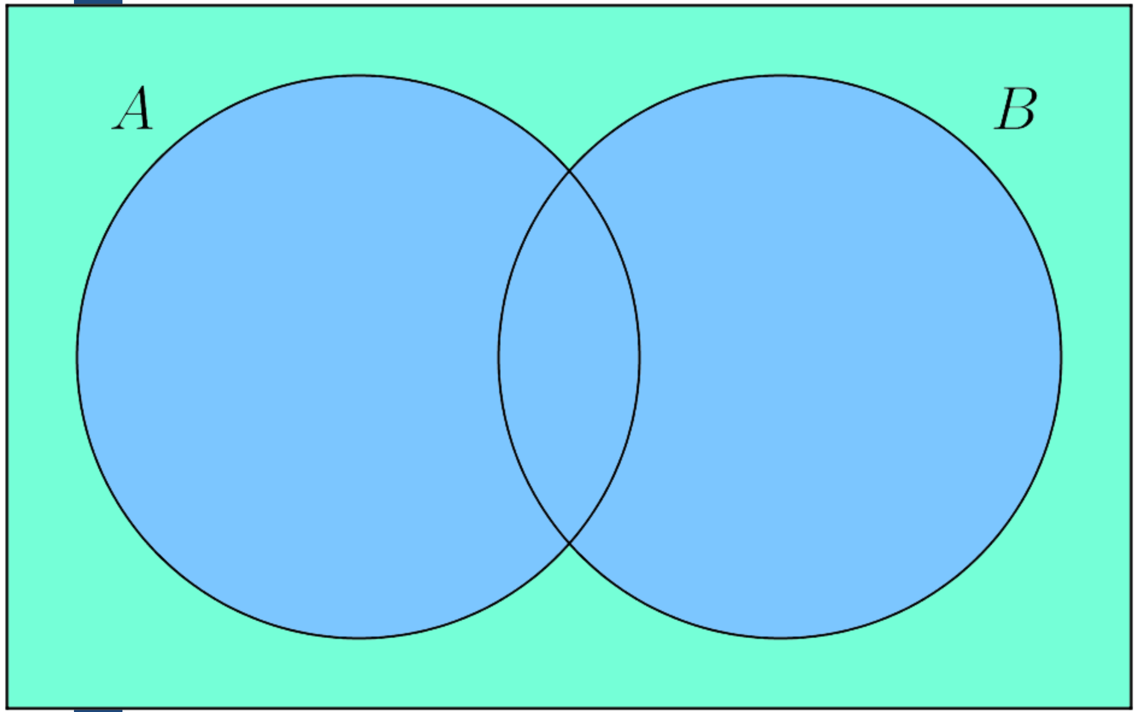
Date: 23rd March 2021

VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS

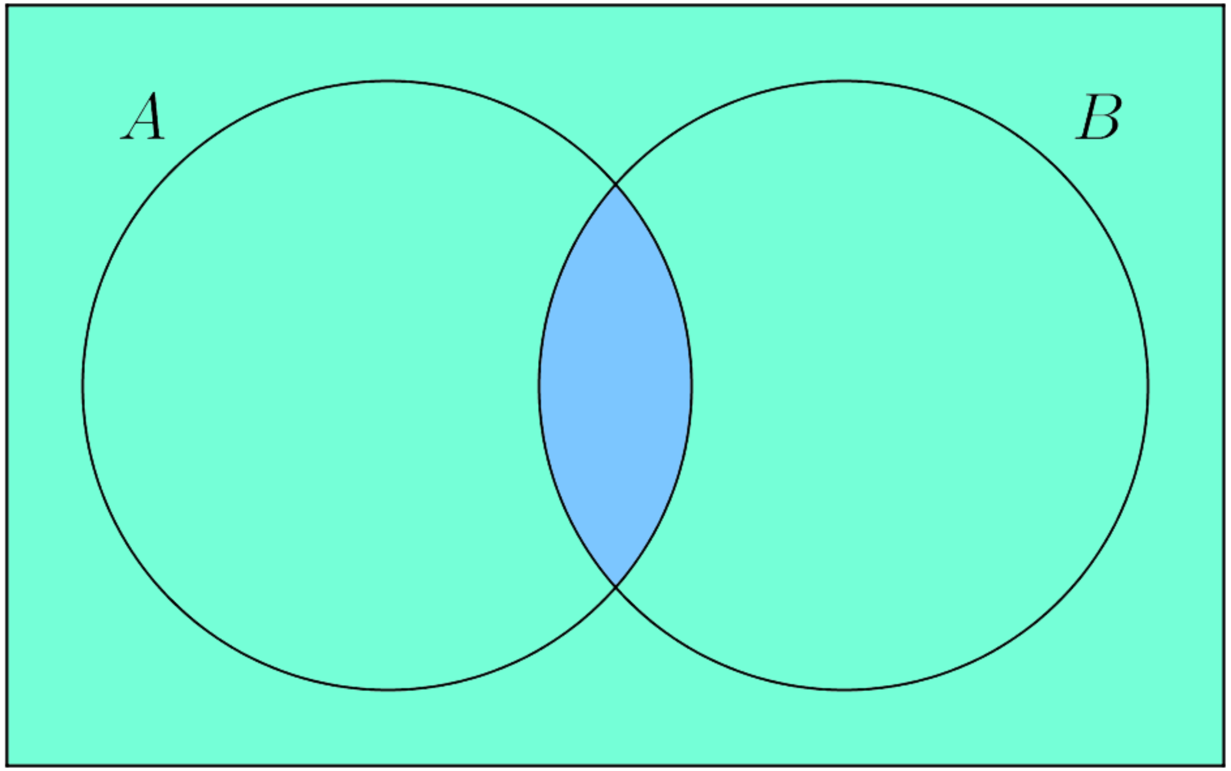
Faculty: CA Arijit Chakraborty

De Morgan's Law



$A \cup B$

$(A \cup B)^c = A^c \cap B^c$



$A \cap B$

$(A \cap B)^c = A^c \cup B^c$

Probability distributions

- If we **measure a random variable many times, we can build up a distribution of the values it can take.**
- Imagine an underlying distribution of values which we would get if it was possible **to take more and more measurements under the same conditions.**
- This **gives the probability distribution for the variable.**
- A probability distribution also possesses all the characteristics of an observed distribution. : **mean median, mode, standard deviation etc.**

Continuous probability distributions

- Because continuous random variables **can take all values in a range, it is not possible to assign probabilities to individual values.**
- Instead we have a **continuous curve, called a probability density function**, which **allows us to calculate the probability a value within any interval.**
- **This probability is calculated as the area under the curve between the values of interest. The total area under the curve must equal 1.**
- **Continuous probability distributions = Normal Distribution**

16.9 RANDOM VARIABLE - PROBABILITY DISTRIBUTION

- **For example, if a coin is tossed three times and if X denotes the number of heads, then X is a random variable. In this case, the sample space is given by**
- **S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} ,** and we find that X = 0 if the sample point is TTT
- X = 1 if the sample point is HTT, THT or TTH   X = 2 if the sample point is HHT, HTH or THH and X = 3 if the sample point is HHH.
- **Distinction between a discrete random variable and a continuous variable.**
- A random variable defined on a **discrete sample space is known as a discrete random variable and it can assume either only a <u>finite number</u> or a <u>countably infinite</u> number of values.**
- The **number of car accident, the number of heads etc. are examples of discrete random variables.**
- A continuous random variable, **like height, weight etc. is a random variable defined on a continuous sample space and assuming an <u>uncountably infinite number</u> of values.**

Odds in favour of event A

- **The ratio of no. of favourable events to the no. of unfavourable events is known as odds in favour of the event A and its inverse ratio is known as odds against the event A.**
- For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.
- i.e. P (A U B)
- or P(A + B) = P(A) + P(B)
- For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.
- i.e. P(A U B) = P(A) + P(B) – P(A Intersection B)
- For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred
- i.e. P(A intersect B) = P(A) × P(B/A)

- ## 16.10 EXPECTED VALUE OF A RANDOM VARIABLE
- *Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities. Hence, if a random variable x assumes n values x1, x2, x3 ...., xn with corresponding probabilities p1, p2, p3 ...., pn , where pi's satisfy (16.33) and (16.34), then the expected value of x is given by*
- **☐ = E(x) = ☐ pi xi (16.41)**
- Expected value of x2 in given by
- **E(x2) = ☐ pi xi^2 .... (16.42)**
- Variance of x, to be denoted by , ☐2 is given by
- **V(x) = (sigma $)^2$ = E(x – u $)^2$ (Here u= mew , mean)**
- **E(x^2) -- u ^{2}**
- The **positive square root of variance is known as standard deviation and is denoted by sigma (small)**

- If y = a + b x, for two random variables x and y and for a pair of constants a and b, then the mean i.e. **expected value of y** is given by
- ***mew* y = a + b mew x** (16.45)

- and the **standard deviation of y** is
- **Sigma y = Mod b * sigma x** ……. (16.46)

- 1. **Expectation of a constant k is k**
- **i.e. $E(k) = k$ for any constant k.**
- 2. Expectation of sum of two random variables is the sum of their expectations.
- i.e. **$E(x + y) = E(x) + E(y)$ for any two random variables x and y**
- 3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.
- i.e. **$E(k x) = k.E(x)$ for any constant k (16.53)**
- Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.
- i.e. **E(xy) = E(x) ☐ E(y)**

- **Example 16.29:** In a business venture, a man can make a profit of ` 50,000 or incur a loss of ` 20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?
- **Solution:** If the profit is denoted by x, then we have the following probability distribution of x: X : ` 50,000 ` –20,000
- P : 0.75 0. 25
- Thus his expected profit E(x) = p1x1 + p2 x2
- = 0.75 □ ` 50,000 + 0.25 □ (` – 20,000)
- = ` 32,500

- **Example 16.32**: A number is selected at random from a set containing the first 100 natural numbers and another number is selected at random from another set containing the first 200 natural numbers. What is the expected value of the product?
- **Solution:** We denote the number selected from the first set by x and the number selected from the second set by y. Since the selections are independent of each other, the expected value of the product is given by
- E(xy) = E(x) ☐ E(y)(1)
- Now x can assume any value between 1 to 100 with the same probability 1/100 and as such the probability distribution of x is given by
- P = 1/100 1/100 ……
- X= 1 2 …..
- E(x) = 1/100 * n (n+1)/ 2 = 101/ 2
- Similarly , E(y)= 201/2
- E(xy) = 101/2 * 201/2 = 5075.25

THANK YOU